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Equity and the Sales Ratio Study in Vermont

Testimony to the House Ways
and Means Committee



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Executive Summary

I had concerns with the current methodology of the ratio study: with small sample sizes, the impact of The Law of the Vital Few, horizontal & vertical equity Statewide, insufficient stratification in testing model performance, and Vermonters are being taxed fairly and equitably. After an extensive literature review and attending three national/international conferences, a presentation by the Lincoln Institute for Land Management presented a new tool to help assessors to determine horizontal & vertical equity within their jurisdiction. Using the Fiscal Year 2022 dataset 2 variables were prepared for analysis Equalized Listed Value (LV/CLA) and Sales Price Real (PREAL). This is testing the results of equalization from the ratio study. The following Vertical Equity Report performed 5 tests for vertical equity “**indicating regressivity**” and the test for horizontal equity was **out of standard**. This report is insufficient to answer all the questions raised, but this supports greater discussion on what standards should be used. Professional suggestions were made to help alleviate the issue raised with the ratio study. The most important recommendation is **implementing the solutions made by the IAAO Consulting Service in Pub. RP-1348. *Ratio Study and Equity Tools in Vermont***, from VALA Newsletter Winter 2025 edition was attached with this document which goes into greater detail on this subject.

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The Problem

I am a senior instructor, mentor, and Advocacy Task Force member for the IAAO. I extensively teach classes in the 300 series on mass appraisal and statistics. I have had concerns about the current methodology used for the ratio study. I tried to brainstorm questions I would like to answer. The reason is to bring these answers back to the Legislature to help inform and affect policy. They were:

- The issues raised with H.480 and Act 68 (2023) may not be answered in a complete way due to how the questions were formulated for the research project.
- With the current ratio study, small sample sizes may have an impact on representativeness within municipality. This would be represented by the Law of the Vital Few (Pareto Principle). As an example, 30 sales are used as a sample to represent a municipality with 800 properties.
- The possible issues with horizontal and vertical equity Statewide and appropriate testing not being performed.
- Insufficient stratification to analyze problems between assessment models and market movement with the lack of a market adjustment for time for sales prices. The potential of the current Category system does not provide adequate stratification for analysis. The current methodology does not use CAMA variables for stratification.
- Is there a false sense of confidence with the current methodology?
- Are Vermonters assessed fairly and equitably?

Literature Review and Interviews

I performed an extensive literature review which is outlined in *Ratio Study and Equity Tools in Vermont*, VALA Newsletter Winter 2025 edition, attached with this document. I also attended 3 national/international conferences in order to ask experts in the fields of Economics, Governance, Statistics, Artificial Intelligence, Mass Appraisal, and Assessment about the best approach to solve the problems raised.

I saw a presentation of Ron Rakow and Paul Bidanset Ph.D.(c) from the Lincoln Institute of Land Management, titled “Assessment Tools and Techniques to Measure and Diagnose Issues with Vertical Equity” at the International Research Symposium for International Property Tax Institute (IPTI) & IAAO in Amsterdam, NL. They presented the Lincoln Institute Vertical Equity App (<https://www.lincolninst.edu/data/lincoln-institute-vertical-equity-app/>). This was a tool to test horizontal and vertical equity only using two variables to assess values and sales price.

Methodology

I have access to the Fiscal Year 2022 dataset (VTSalesStudyDataSetAsOf07112022) using table PVR_PTR1 with 51,072 records spanning 4/2/2018 to 3/31/2021. I downloaded from the PVR website Fiscal Year 2022 Common Level of Appraisal (CLA) calculations spanning June 29 to August 18, 2021. The CLA, the List Value (LV) variable is directly from the Grand Lists for the municipalities. I was not testing the performance to the assessment models. I wanted to test how well the ratio study equaled the sales prices in the study. I performed a transformation to create Equalized Listed Value (LV/CLA). I chose Sales Price Real (PREAL) variable. This has removed excessive personal property from the study. I need to stress:

- I treated the dataset at face value,
- I did not perform any sales validation or market adjustment for time,
- Approximately one year of the three-year period was impacted by a COVID-19 real estate market, and
- An A/SR was calculated only for trimming purposes and was not used by Lincoln Institute Vertical Equity App.

It is typical in assessment analysis to have a 95% confidence interval. For sound appraisal judgement, I would trim 5% to limit issues in analysis. The records that were trimmed were as follows: 7 for no SPAN, 409 for being less than \$10,000, and 1,068 from both high and low A/SR. This left 48,522 records for analysis, which I deemed sufficient.

Summary of Results

After uploading LV/CLA and PREAL within the website I was presented with a summary report that is attached to this document. There were 5 tests for vertical equity and 1 test for horizontal equity. They were Assessment Ratio Analysis (ASR), Coefficient of Dispersion (COD), Price Related Differential (PRD), Coefficient of Price Related Bias (PRB), Spearman's Rank-Order Correlation (Spearman's rho), and Gini Measures (Gini). For a definition of regressivity/progressivity can be found under **Measures of Uniformity** section on page 15 in this document by (Carter 2016).

- **ASR** – “The range between median ASR for the lowest priced decile (1.628) and highest (0.923) decile is 0.705... a **regressive trend**.”
- **COD** – “The COD of the properties provided is 36.805.” (5.0 to 20.0) **Out of standard on horizontal equity**
- **PRD** – “for the properties provided is 1.176, which is outside the acceptable range of .98 to 1.03 recommended by the IAAO and **indicates regressivity** is present.

- **PRB** – “for the properties provided is -0.132, which is outside the IAAO recommended range of -0.05 – 0.05 and **indicates regressivity** is present.”
- **Spearman’s rho** – “of -0.408 suggests a high correlation of the ASR and sales ranking, which is confirmed in the plot of these data. The asymptotic T-statistic approximation suggests this correlation is statistically significant at a 0.001 confidence level. As such, the Spearman test **indicates regressivity.**”
- **Gini measures** – “for this distribution provides a KI of -0.058 and a MKI of 0.869 **indicating regressivity** in the distribution.”

Fiscal Year 2022 appears to have issues with both Horizontal and Vertical Equity. This is only one test, and it does not definitively answer the questions raised in the Problem section of this report. This does open an opportunity for further reflection on how the assessment process and the ratio study are performed and at what level of standard.

Possible Solutions

Here are some of my professional recommendations that might help with the issues raised.

- The Ratio Study should be front and center in decision making on how we perform assessments in Vermont,
- Five or six assessment districts (~54,600 to 65,500 parcels) should be examined, which takes advantage of The Law of Large Numbers,
- The ratio studies are created by the district using CAMA variables and audited by PVR,
- Market Adjustment (Time) on Sales Prices this captures movement within the market,
- Assessed values to be determined annually this relies on the Sales Comparison Approach over the Cost Approach which can lead to greater acceptance. Relying on experienced personnel over the use of AI tools.
- Test for vertical and horizontal equity Statewide with independent audits,
- Implement a continuous improvement program to stay current with best practices, and
- **Implement the solutions made by the IAAO Consulting Service in Pub. RP-1348.**

Vertical Equity Report

Lincoln Institute of Land Policy

This report provides guidance on the vertical equity of the assessed values for the file that was provided. Vertical equity for assessments is defined as: “differences in the levels of assessment of properties related to the value ranges of the properties, i.e., properties of higher value have assessment levels different from properties of lower value.” Vertical equity exists when assessment levels remain consistent across the entire range of sales prices.

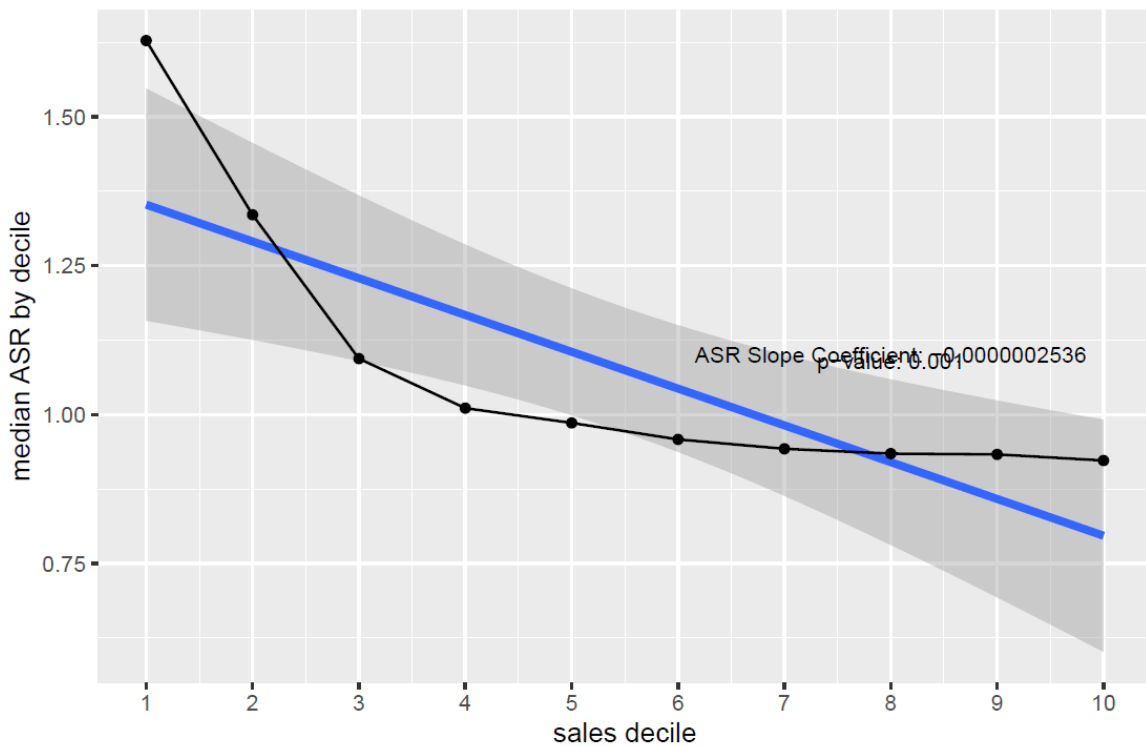
Detailed guidance on sales ratio studies is provided by International Association of Assessing Officers Standard for Sales Ratio Studies (Standard). Measuring the level of vertical equity of assessments is a complex task since the market value of any given property is unknown. As a result, the notion of a single or best measure of vertical equity may be illusive. Instead, the best approach may be to calculate and evaluate all the current measures in the Standard, with the addition of Gini measures which are new to the assessment profession, to develop a more complete picture of the level of vertical equity for a given distribution. The following analysis is provided based in part on the Standard.

Assessment Ratio Analysis

The first step in this analysis is an analysis of the assessment to sale ratio which is calculated by dividing a property’s assessment by its sale price. The overall assessment level is determined by the median assessment to sale ratio (ASR). The median ratio is preferred because it is less affected by extreme values than other measures of central tendency, such as averages. **The overall median ASR is 1.008.**

The median ASR can also be used to evaluate vertical equity when calculated for each sale price decile as provided in this following chart. The chart displays how the ASR changes as the sales price increases. **The range between median ASR for the lowest priced decile (1.628) and highest (0.923) decile is 0.705.** This suggests a trend toward lower ratios as sales prices increase, a **regressive trend.**

Median ASR by Sale Decile



Coefficient of Dispersion

The Coefficient of Dispersion, or COD, is not considered to be a measure of vertical equity. It does, however, provide important information on the uniformity or variability of the distribution, and therefore can be informative in this context.

The COD of the properties provided is 36.805. IAAO guidance for the COD are provided below for various residential property uses.

IAAO COD Ranges by Property Type

Type of Property - General	Type of Property - Specific	COD Range
Single-family residential (including residential condominiums)	Newer or more homogeneous areas	5.0 to 10.0
Single-family residential	Older or more heterogeneous areas	5.0 to 15.0
Other residential	Rural, seasonal, recreational, manufactured housing, 2-4 unit family housing	5.0 to 20.0
Income-producing properties	Larger areas represented by large samples	5.0 to 15.0
Income-producing properties	Smaller areas represented by smaller samples	5.0 to 20.0
Vacant land		5.0 to 25.0

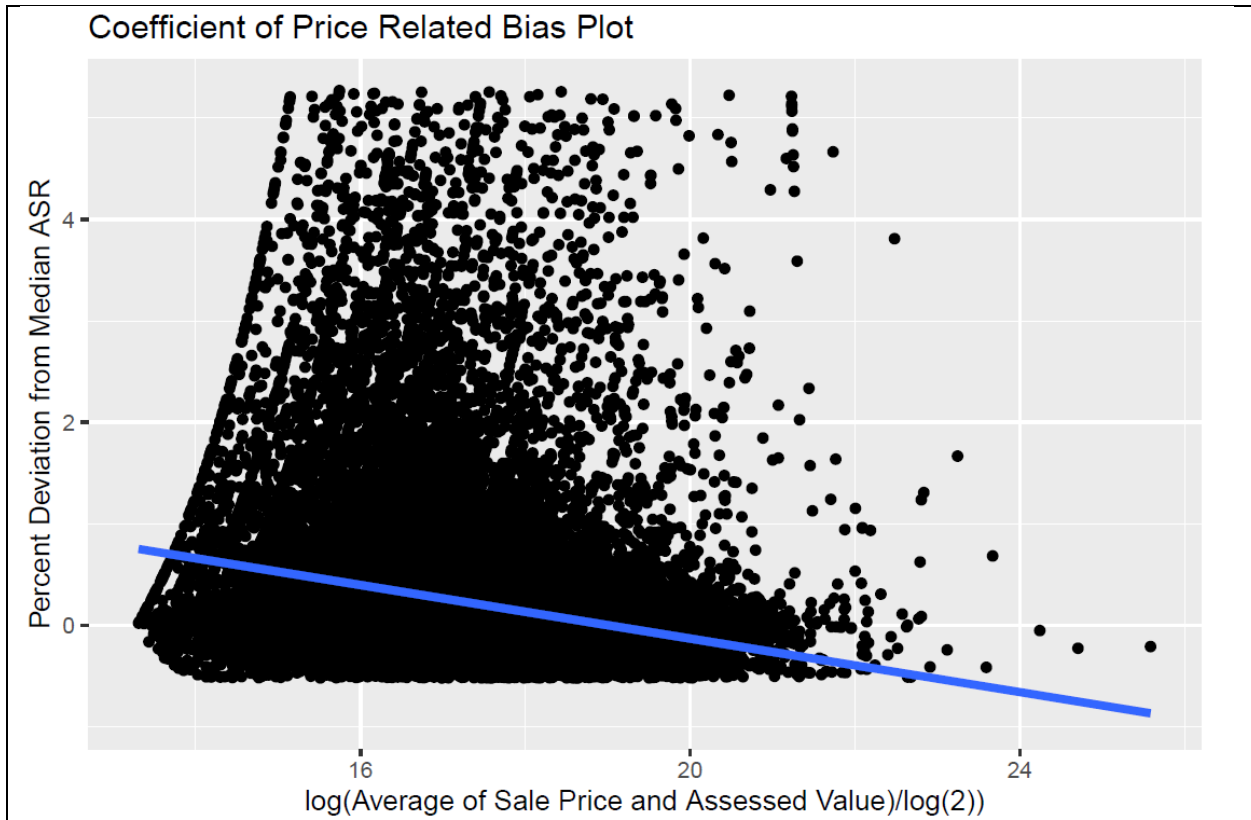
Type of Property - General	Type of Property - Specific	COD Range
Other real and personal property		Varies with local conditions

Price Related Differential

The Price Related Differential, or PRD is an index statistic used for measuring vertical equity. The PRD is calculated by dividing the mean ASR by the weighted mean ASR. If vertical equity exists, this ratio should be close to 1.00. A PRD above 1.00 suggests a regressive distribution where lower-valued properties are assessed at a higher level than higher-valued properties. A PRD below 1.00 suggests a progressive distribution, where the reverse is true. The PRD as a vertical equity measure is useful but somewhat flawed. The PRD's strengths are its simplicity and ease of calculation. A disadvantage of the PRD is that it can be distorted by a small number of very high-priced properties that can lead to an indication of regressivity when none is present. As a result, the PRD can be a useful indicator that more analysis is necessary, but by itself it is not necessarily conclusive that vertical inequity exists. **The PRD for the properties provided is 1.176, which is outside the acceptable range of .98 to 1.03 recommended by the IAAO and indicates regressivity is present.**

Coefficient of Price Related Bias

The Coefficient of Price Related Bias, or PRB, measures vertical equity with an index calculated by regression that quantifies the relationship between ASRs and value in percentage terms. If vertical equity exists, the PRB value should be close to zero. A PRB below zero suggests a regressive distribution where lower-valued properties are assessed at a higher level than higher-valued properties. A PRB above zero suggests a progressive distribution, where the reverse is true. The PRB is a more robust measure that is not as susceptible to the influence of outliers as the PRD. **The PRB for the properties provided is -0.132, which is outside the IAAO recommended range of -0.05 – 0.05 and indicates regressivity is present.**



Spearman’s Rank-Order Correlation

While not specifically discussed in the section of the Standard on vertical equity, the Spearman Rank test is mentioned in the Test of Hypotheses section of the Standard. Specifically, Spearman’s Rank is recommended for evaluating whether low and high prices properties are appraised at equal percentages of market value i.e., whether vertical equity exists.

The Spearman’s Rank measures the strength and direction of association between two ranked variables. Ranks are determined for each property in the analysis, one rank for sale price, the other for assessment to sale ratio (ASR). A correlation analysis of the two ranks is performed, with the resulting statistic referred to as Spearman’s rho. A Spearman’s rho of close to 0 demonstrates a low correlation between ASR and sales price and suggests vertical equity is present. A statistical test (t-test) is also performed to determine if the correlation is statistically significant. The distribution of a Spearman’s rank is also plotted to provide a visual representation of the analysis.

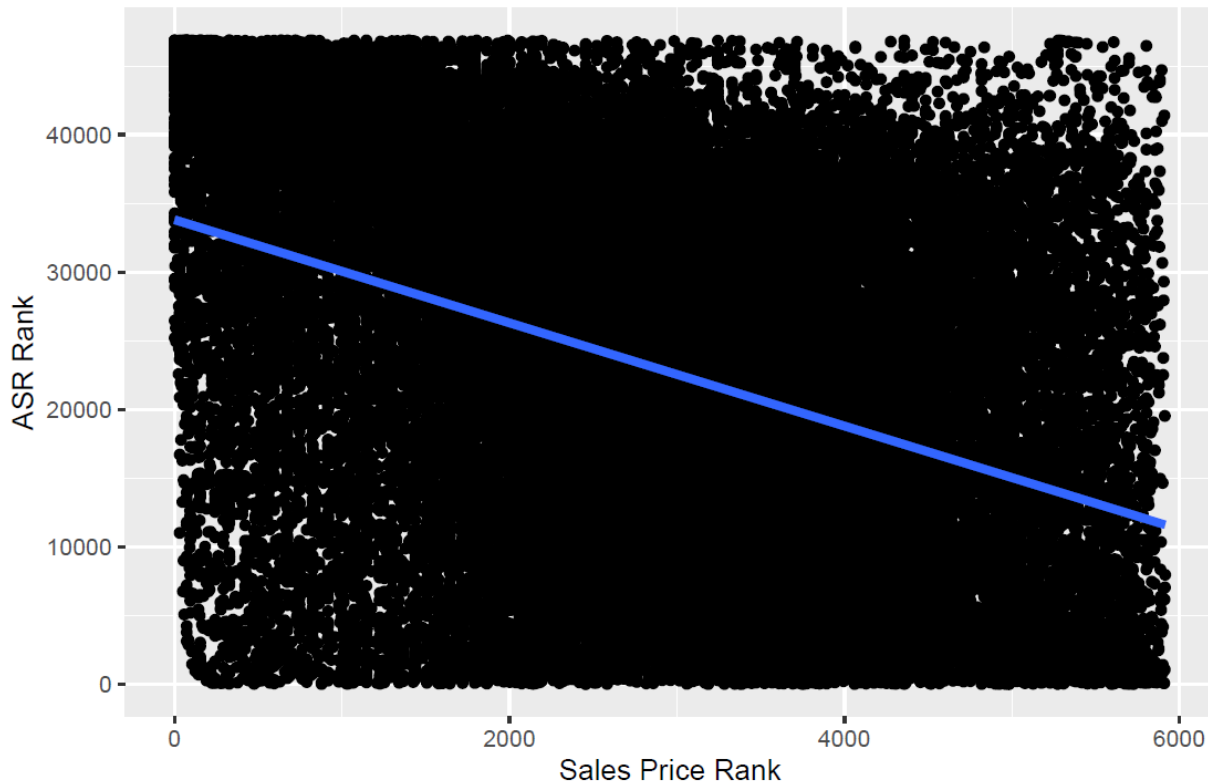
While the Spearman analysis can provide useful information on the relationship between ASR and sales, its limitation is that it is a simple correlation that lacks other important information on the distribution. **A Spearman’s Rho of -0.408 suggests a high correlation of the ASR and sales ranking, which is confirmed in the plot of these data. The asymptotic T-statistic approximation suggests this correlation is statistically significant at a 0.001 confidence level. As such, the Spearman test indicates regressivity.**

Spearman’s Rho Ranges

Spearman’s Rho Coefficient	Interpretation	Acceptable Range
-1.0 to -0.9	Very Strong Regressive Correlation	No
-0.89 to -0.70	Strong Regressive Correlation	No
-0.69 to -0.40	Moderate Regressive Correlation	No

Spearman's Rho Coefficient	Interpretation	Acceptable Range
-0.39 to -0.20	Weak Regressive Correlation	Yes
-0.19 to 0.01	Very Weak Regressive Correlation	Yes
0	No Correlation	Yes
0.01 to 0.19	Very Weak Progressive Correlation	Yes
0.20 to 0.39	Weak Progressive Correlation	Yes
0.40 to 0.69	Moderate Progressive Correlation	No
0.70 to 0.89	Strong Progressive Correlation	No
0.90 to 1.0	Very Strong Progressive Correlation	No

Spearman's Rank-Order Correlation Plot

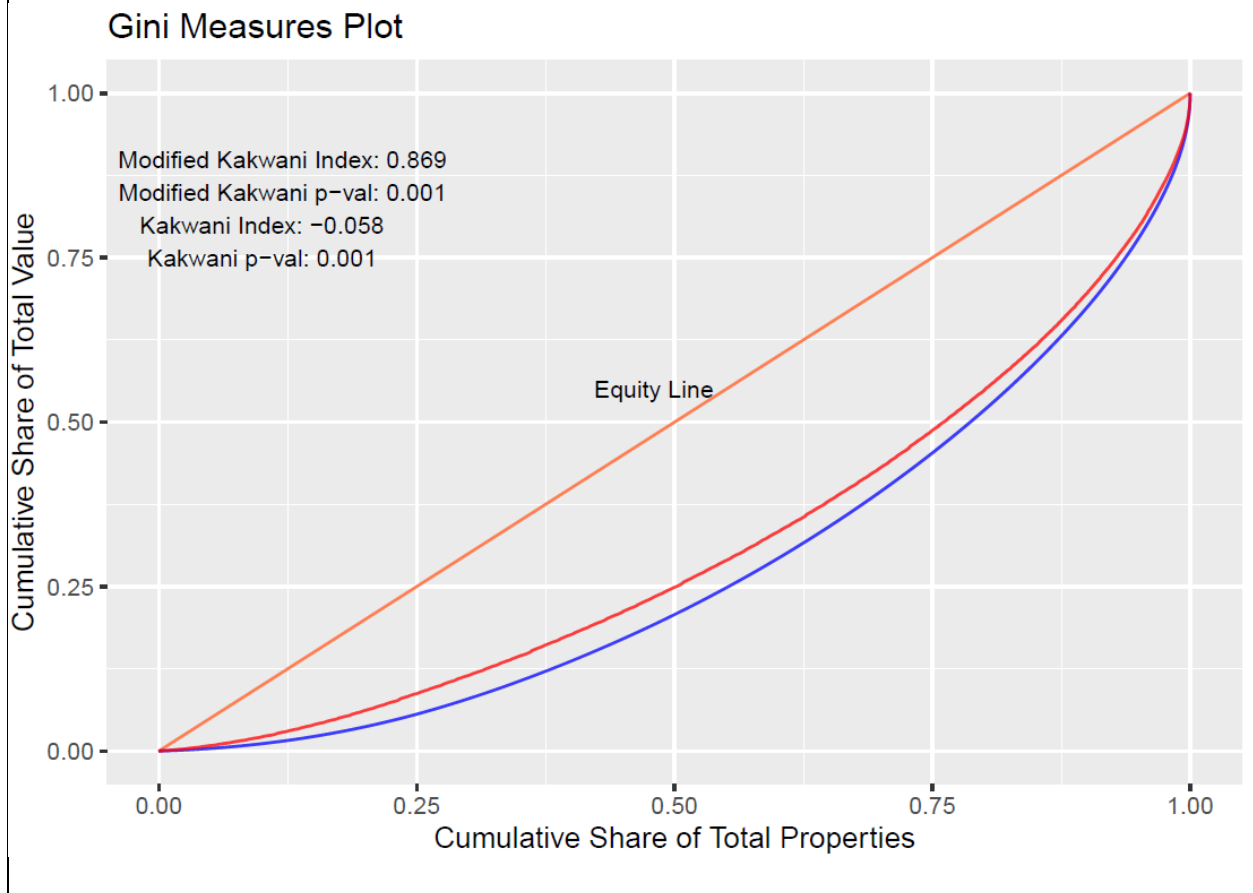


Gini Measures

While the use of Gini measures as a method for measuring inequity has been common in economic analysis for decades, it has only recently been considered as a measure of vertical equity for assessments. More information on the use of Gini measures to measure vertical equity in assessments can be found in recent papers by Quintos (2020), and McMillen and Singh (2021).

Gini measures are like Spearman's rank in that they do not use sales ratios to determine equity and that both measures use a ranking of variables in their calculation. However, Gini's go beyond a simple measure of co-movement as is done in Spearman's Rank, quantifying how the distribution of assessments behave relative to the distribution of sales at given price levels. By capturing the cumulative distribution behavior of assessments and prices across ordered price levels, Gini measures provide more information on the vertical equity of the entire distribution. The Gini analysis provides two numerical measures that summarize the relationship of the sales and assessment distributions, the Kakwani index (KI) and the Modified Kakwani index (MKI). Assessments are considered regressive when KI is less than zero, or MKI is less than one. A progressive distribution occurs when KI is above zero, or MKI is above one.

In addition to these indexes, the level of equity in assessments can also be visualized in a Gini analysis via a plot of the relationship between of the sales and assessments. If the lines move together, vertical equity is present. Where the line representing the assessments lies above the sales line, a regressive distribution is indicated. Where the assessment line is below the sales line, a progressive distribution is implied. Unlike the other vertical equity measures which provide a global measure of vertical equity for the entire distribution, the Gini plot provides an indicator of the measure of equity across the entire price distribution. This information can be used to better understand the performance of a valuation model across different segments of the price distribution, allowing for better diagnosis of potential vertical equity issues. The Gini analysis for this distribution provides a **KI of -0.058** and a **MKI of 0.869** indicating **regressivity** in the distribution. This finding is confirmed through the inspection of the lines representing assessments and sales produced by the Gini analysis.



Article – VALA Newsletter Winter 2025 edition

Ratio Study and Equity Tools in Vermont

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With all the potential changes to our industry from the legislature, I wanted to take a moment and take a deep dive into equity in Vermont with some new tools that can help. This article is based on a larger paper that was presented at the National Tax Association conference in Detroit. The major concern raised was potential issues with vertical inequity in Vermont's assessment system in H.480 and Act 68 (2023). This can be a daunting technical subject, but if given patience there is a possible solution. The International Association of Assessing Officers (IAAO) Research and Standards Committee is currently reviewing and updating the *Standard on Ratio Studies* (2013). Recommendations on Vertical Equity are being examined in order to improve performance.

Time Adjustment and Ratio Studies

The *ad valorem* Appraisal Level, also known as Assessment to Sales Ratio (A/SR), is the Assessed Value (AV) divided by the Sales Price (SP), which is the ratio used for the Ratio Study. According to IAAO (2013, page 8):

"Local jurisdictions should use ratio studies as a primary mass appraisal testing procedure and their most important performance analysis tool. The ratio study can assist such jurisdictions in providing fair and equitable property assessment. Ratio studies provide a means for testing and evaluating mass appraisal valuation models to ensure that value estimates meet attainable standards of accuracy... Ratio studies also play an important role in judging whether constitutional uniformity requirements are met."

The advantage of ratio analysis is that it can be performed on **any strata** of the Real Property Market. The State places a lower limit on AV for the Ratio Study of \$10,000. A small difference in negotiation on SP can have a large difference in A/SR on properties with

smaller values. The modeler must take extra care to trim sales but keep representativeness for this stratum of the market. In IAAO (2013, p. 17), the Level of Appraisal:

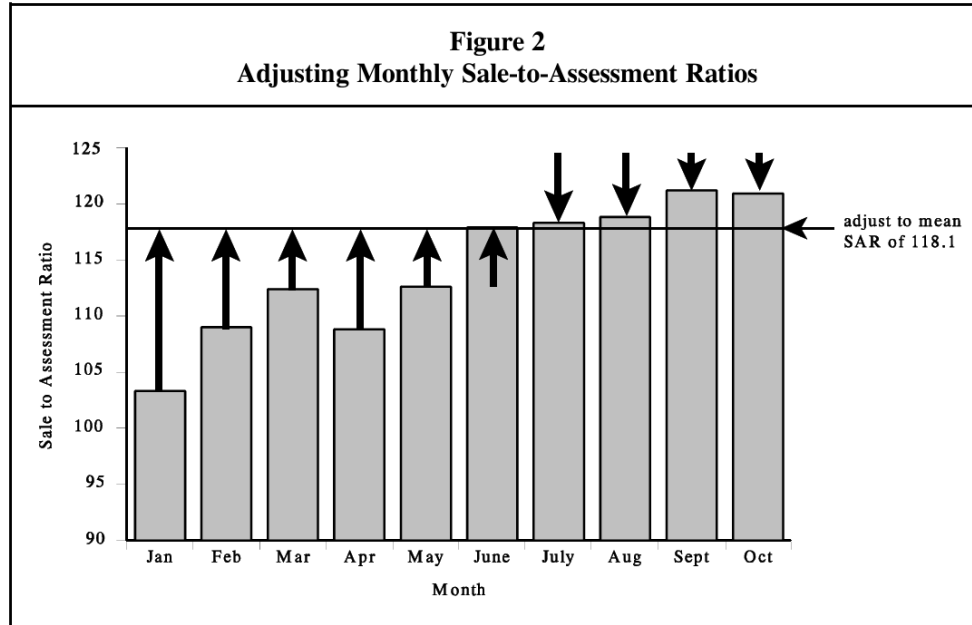
“In analyzing appraisal level, ratio studies attempt to measure statistically how close appraisals are to market value... While the theoretically desired level of **appraisal is 1.00**, an appraisal level **between 0.90 and 1.10 is considered acceptable for any class of property**. However, each class of property must be **within 5 percent of the overall level of appraisal of the jurisdiction... Both criteria must be met**. By themselves, the calculated measures of central tendency provide only an indication, not proof, of whether the level meets the appropriate goal.”

Dornfest (2024) mentioned two time-adjustment methods in the presented Excel template: a Linear Trend and an Exponential Trend. This was calculated using a predefined formula in Excel. Gloude-mans (1999, pages 263-268) discusses using a Sale Price to Assessment Ratio (S/AR) to create a linear time adjustment. This method was reinforced by the University of British Columbia (UBC) (2003, pages 1-7) since the monthly mean ratio would represent the sales on the 15th of the month in question. One could create a trended monthly factor to the statutory Date of Appraisal, April 1.

Figure 1 – Time Adjustment Trend Factors

MONTH	AVERAGE	CALCULATION	FACTOR
1 (Jan)	103.4	118.1 ÷ 103.4	1.142
2 (Feb)	109	118.1 ÷ 109.0	1.083
3 (Mar)	112.4	118.1 ÷ 112.4	1.051
4 (April)	108.9	118.1 ÷ 108.9	1.084
5 (May)	112.6	118.1 ÷ 112.6	1.049
6 (June)	117.9	118.1 ÷ 117.9	1.002
7 (July)	118.3	118.1 ÷ 118.3	0.998
8 (Aug)	118.8	118.1 ÷ 118.8	0.994
9 (Sept)	121.2	118.1 ÷ 121.2	0.974
10 (Oct)	120.9	118.1 ÷ 120.9	0.977

Figure 2 – Applying Time Adjustment Trend Factors



Real property markets do not necessarily have linear growth and/or decline. This methodology best captures the cyclical nature of a real property market. This analysis uses the mean instead of the median ratio since real property markets tend to be non-parametric. We want to limit the influence of outliers and extremes. With the limited number of sales in some towns, there were less than 30 valid arm's length transactions in three years for all categories. Recommended time adjustments would be calculated on a regional or statewide basis with an adequate number of sales.

Common Level of Appraisal (CLA)

The CLA was created out of Vermont Act 60 (1997). It is calculated to equalize the Education Grand List for school funding purposes. Act 68 (2023) removed the CLA as a reappraisal trigger. Vermont Legislative Joint Fiscal Office (2022) goes into sufficient detail to explain how the CLA is calculated. The Ratio Study uses the Aggregate Ratio, the Weighted Mean Ratio (See PRD formula page 5), per Category to calculate CLA. The advantage is not affected by the inherent upward bias as the Mean Ratio.

A Grand List Category can fail if there are less than five records or Sampling Error greater than 15%. When this is true, the Sales Study uses the Class Ratio. If the Class Ratio fails, then it defaults to the Town Ratio. The "Other" category is user-defined and homes around a community feature. It can be from any category, but it needs to be defined according to what Class it is in for analysis. The ratio study is meant to be used to test different strata of the market that may be out of standard. Traditionally it is not limited to only category, class,

and town. It is compared to the variables within your CAMA database. Some examples of ratio vs variables may be; story height, fixture count, square footage GLA, by sales price range, acreage, etc. In Massachusetts, municipalities perform their own in-depth ratio study, in compliance to statute, and is audited by the State oversight agency.

Figure 3 – Example alternative Ratio Study stratification

CATEGORY	SINGLE FAMILY PROPERTIES BY PRICE RANGES \$								
	NUMBER	MEDIAN	MEAN	AGG MEAN	COD	COV	PRD	LOW CL	UPP CL
\$100,000 OR LESS	125	0.930	0.908	0.910	10.7	14.2	99.7	0.898	0.941
\$100,001 - \$200,000	522	0.945	0.949	0.949	8.2	10.9	100.1	0.933	0.953
\$200,001 - \$300,000	282	0.960	0.962	0.961	8.9	11.2	100.1	0.945	0.983
\$300,001 - \$400,000	76	0.952	0.974	0.975	11.3	13.7	99.9	0.899	1.019
\$400,001 OR MORE	31	0.983	1.005	1.010	11.4	14.4	99.4	0.937	1.086

If there is insufficient sales volume in a municipality for the last 3 years, PVR staff can insert appraisals in lieu of sales. The concern about adding appraisals is whether they truly represent the market and could lead to unintentional bias entering the Sales Study.

Measures of Uniformity

Due to the rural nature of the majority of Vermont's real property market, a smaller number of sales in a municipality can occur. To combat this, the State uses three years of validated sales for the ratio study for analysis. The best definition to describe the two types of bias in assessment appraisals is from Carter (2016, page 3), where the author found.

“...horizontal inequity, concerns price differentiation between homes with the same attributes. If two homes with the same attributes (including geographical location) are assessed at different values, horizontal inequity exists. Generally, more concerning, however, is vertical inequity. Vertical inequity exists when the ratio of assessed value to sale value changes over the value of properties generally, and it can be either regressive (where low-value homes are regularly assessed at greater ratios to the sale price than the high-value homes in the sample) or progressive (the opposite phenomenon).”

The primary test for horizontal equity is the Coefficient of Dispersion (COD) from IAAO (2013, page 13). COD currently is the only reappraisal trigger derived from the ratio study. Act 68 (2023) raises issues of vertical inequity. A test for Vertical Equity is the Price Related Differential (PRD). There are issues with this test which we will discuss later in this article.

Figure 4 – Coefficient of Dispersion (COD)

is:

$$(1) \text{ COD} = \frac{100}{R_m} \left(\frac{\sum_{i=1}^N |R_i - R_m|}{N} \right)$$

where:

COD = coefficient of dispersion, i.e., the average percent of dispersion around the median assessment ratio;

R_m = median assessment ratio;

R_i = observed assessment ratio for each parcel;

N = number of properties sampled.

Figure 5 – Price Related Differential (PRD)

$$(3) \text{ IR} = \frac{\sum_{i=1}^i \sum_{j=1}^j \left(R_{ij} \bullet \frac{w_j}{w} \right)}{N} = \frac{\sum_{i=1}^i \sum_{j=1}^j \left(ASV_{ij} \bullet \frac{w_j}{w} \right)}{\sum_{i=1}^i \sum_{j=1}^j \left(APV_{ij} \bullet \frac{w_j}{w} \right)}$$

} Simple Mean

} Weighted Mean

where:

N = the total number of sampled properties;

i = the sampled parcel;

j = the stratum;

w_j = the weight of every sampled parcel drawn from the jth stratum (see previous discussion of sample weighting in relation to COD);

$\frac{w_j}{w}$ = the total number of parcels in a stratum divided by the total number sampled in that stratum (see previous discussions of sample weighting in relation to COD);

R_{ij} = ratio of assessed value to estimated market value (appraisal or sale) (one for each sampled property in each stratum);

ASV_{ij} = assessed value of the "ith" sampled property in the jth stratum; and

EMV_{ij} = estimated market value of the "ith" sampled property in the jth stratum.

The COD and PRD formulas are from the New York Department of Taxation and Finance (2013, pages B-2, B-5, B-6).

One of the main problems with the PRD from IAAO (2013, page 19) is

"PRDs should be between 0.98 and 1.03. The reason this range is not centered on 1.00 relates to an inherent upward bias in the arithmetic mean (numerator in the PRD) that does not equally affect the weighted mean (denominator in the PRD). When samples are small, have high dispersion, or include properties with extreme values, the PRD may not accurately indicate assessment regressivity or progressivity."

This is reinforced in Gloudeman (2011, page 3).

"the PRD does not provide a meaningful gauge of what it purports to measure. What does a PRD of 1.05 really mean and how much worse is it than PRD of 1.03?... (The Price Related Bias PRB), it measures the percentage relationship between the property values and the assessed ratios and indicates by the percentage of assessment levels that change whenever property values are doubled (or halved)."

Denne (2011, p. 3) supports Gloudeman (2011) with the concerns of the PRD,

"The PRD appears to have risen as a calculation of convenience to accompany U.S. census data that developed for another purpose, serving only to allow users 'to obtain some notion of any association, within a jurisdiction, between assessed values and the property sales price ranges' (U.S. Department of Commerce 1957)...the PRD has several flaws...it does not measure the severity of reported bias in comprehensible terms. It is usually employed to make a strictly yes-or-no compliance decision in a manner akin to testing for statistical significance rather than testing for the magnitude of an effect...the most troublesome aspect of the PRD, however, is its susceptibility to extreme distortion as a result of heteroscedasticity (inconsistent variance among the range of assessment ratios, especially when the variance is systematically increasing with a property value)."

The State Ratio Study does calculate the PRD but has yet to be used. It should be used only to support an additional measure(s) of Vertical Equity. The Price Related Bias (PRB) is an additional test for Vertical Equity. The use of the PRD and PRB together can support each other when determining Vertical Equity, but only if the PRD meets the standard. Typical PRB uses the natural logarithm (2), which is 0.693, Gloudeman (2011, page 5).

Figure 6 - Price Related Bias (PRB)

$$Value = 0.50 \times SP + 0.50 \times (AV \div Median)$$

$$PRB = LN(Value) \div 0.693$$

$$SP = Sales Price$$

$$AV = Assessed Value$$

$$LN = natural logarithm$$

Gloudeman (2011, p. 8) advocates using the PRB to test for Vertical Equity.

"Although the PRD will undoubtedly continue to provide a familiar first line indicator of vertical bias, in the PRB coefficient provides a more meaningful and reliable measure that can either supplement or be used in place of the PRD. Ideally a measure of Vertical Equity would quantify the relationship between the percentage changes in the value and the assessment ratios. The PRB coefficient does so. It also addresses the bias problem, promises the effect of outliers, and measures the statistical significance of any indicated inequities."

IAAO Statistical Tools and Measures Task Force (2023) in section **4. Simulation Results**, the PRB outperformed different statistical tests, but an endogeneity bias was noted in the Conclusions (page 131).

"While more simulation studies that allow for other types of distortions such as outlier effects, nonlinearities, small samples, etc. could produce additional observations and possibly affect our conclusions, the results that would hold true regardless would be that (1) ASR and PRB have at least some endogeneity bias by construction with the ASR likely having more bias than the PRB, ...**We do not recommend one particular test over another but rather that a suite of tests be reported to support the existence or absence of Vertical Equity**".

One of the additional tests for Vertical Equity is the Gini Quotient (Gini) in Quintos (2020, p. 34), it tests equity using shares.

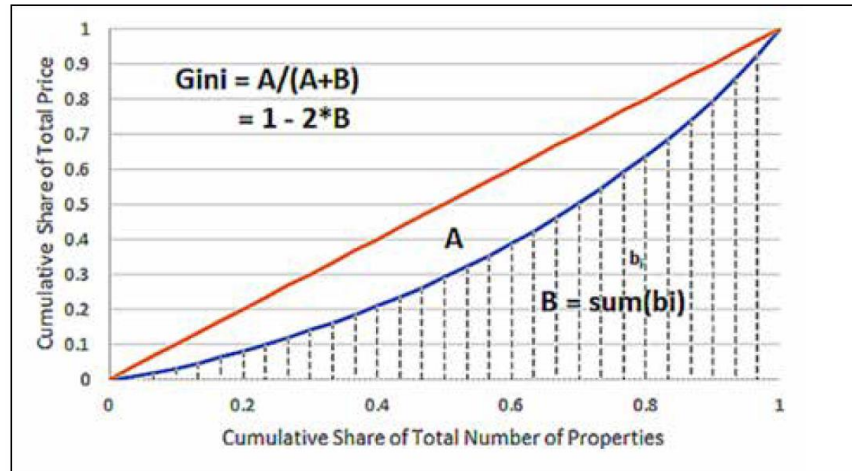
"The tests for Vertical Equity are the Kakwani Index (K.I.) of Kakwani (1977), which is based on the difference of Gini measures between assessment and price... which is based on the ratio. An important distinction between both tests and the PRD and currently used measures is that Gini-based analyses do not use sales ratios... which basically capture the behavior of the appraisal errors."

The three strengths of this method are:

1. The Lorenz Curve is visual by nature,
2. The analysis can be performed in Excel, and
3. Four numbers are generated to explain the relationship between the shareholders.

Figure 7 – Lorenze Curve example

Figure A-1. Calculating the Gini coefficient as areas under a curve



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The PRD and PRB have limitations and should not be used as the sole Vertical Equity test. According to the papers author the Gini requires sufficient sample size should be used for analysis. In the aforementioned Task Force paper recommended a “suite of tests” to be used in order to confirm Vertical Equity.

Lincoln Institute Vertical Equity App

The International Property Tax Institute (IPTI) and the IAAO hosted an International Research Symposium (IRS) in Amsterdam (<https://www.iaao.org/irs-schedule/>) and I was able to attend. I saw a presentation from the Lincoln Institute of Land Management by Ron Rakow and Paul Bidanset, Ph.D.(c) on *Assessment Tools and Techniques to Measure and Diagnose Issues with Vertical Equity*. The Lincoln Institute wanted to give a free tool to help Assessors to test for Vertical Equity (<https://www.lincolnst.edu/data/lincoln-institute-vertical-equity-app/>). Paul Bidanset coded using Python for the statistics with a summary report. The data set that is tested is purged after the report is generated. The tests performed are as follows:

- Median Assessment Ratio Analysis by Decile – this gives a visual method to show stratification within your markets and how close the lower and upper strata ratio's are equitable.
- Coefficient of Dispersion (COD) – the primary measure for horizontal equity.
- Price Related Differential (PRD) – one measure for Vertical Equity. It is best treated as a true/false due to lack of scalability mentioned above.
- Coefficient of Price Related Bias (PRB) – one measure for Vertical Equity. But has issues with endogeneity bias mentioned above.
- Spearman's Rank-Order Correlation (Spearman's rho) – used primarily for non-parametric statistical testing for correlation between variables. One of its strengths Spearman's rho does not rely on ratios and the issues that can be raised. This statistical test is an option for hypothesis testing for nonparametric datasets mentioned in standard, IAAO (2013, page 15).
- Gini Measures (Gini) – this is an additional measure that does not rely on ratios and their possible issues. It works under the principle of shares. The Equity Line represents perfect equity, there are 2 Lorenz Curves: Kakwani Index (KI) and Modified Kakwani Index (MKI). Ideally the KI and MKI curves would be very close to each other. Perfect inequity would be all stakeholders would be 0 and the last stakeholder would be at 100% all the way at the right at 1.00.

I have attached a copy of the *Vertical Equity Report* for Skagit County Washington from an assessment colleague, Assessor Danny Hagen. This shows that the report does work and can be a useful and powerful tool for Vermont. You want to test equalized assessment values vs. sales prices. This is intended to be an apples-to-apples comparison.

Conclusion

To conclude, the methods used to determine Vertical Equity in Vermont need to be examined which tool(s) best improve performance. Time adjustments in ratio studies may be appropriate because real property markets do not always experience linear growth or decline, and outliers need to be limited. The use of additional appraisals instead of sales could introduce unintended bias in a Sales Study which needs to be limited. With reliance on larger sample sizes, the Law of Large Numbers comes into play. This can be a better indicator of the representativeness of the sample. Since only sales within the sample period can be used to determine the assessed value for all properties within the municipality. This can lead to the issue of the Law of the Vital Few, or commonly known as the Pareto Principle if sample sizes are too small. Remember we are performing an estimate of an estimate. Other measures, such as COD, PRD, PRB, Spearman's rho and Gini Quotient have strengths and weaknesses, which suggests that multiple tests should

be used to support Vertical Equity. The Lincoln Institute's Vertical Equity App offers a tool to help assessors, which uses a statistics-driven approach that provides a solution for the issue of Vertical Equity in Vermont.

Figure 8 – Vertical Equity Report for Skagit County Washington

Vertical Equity Report

Lincoln Institute of Land Policy

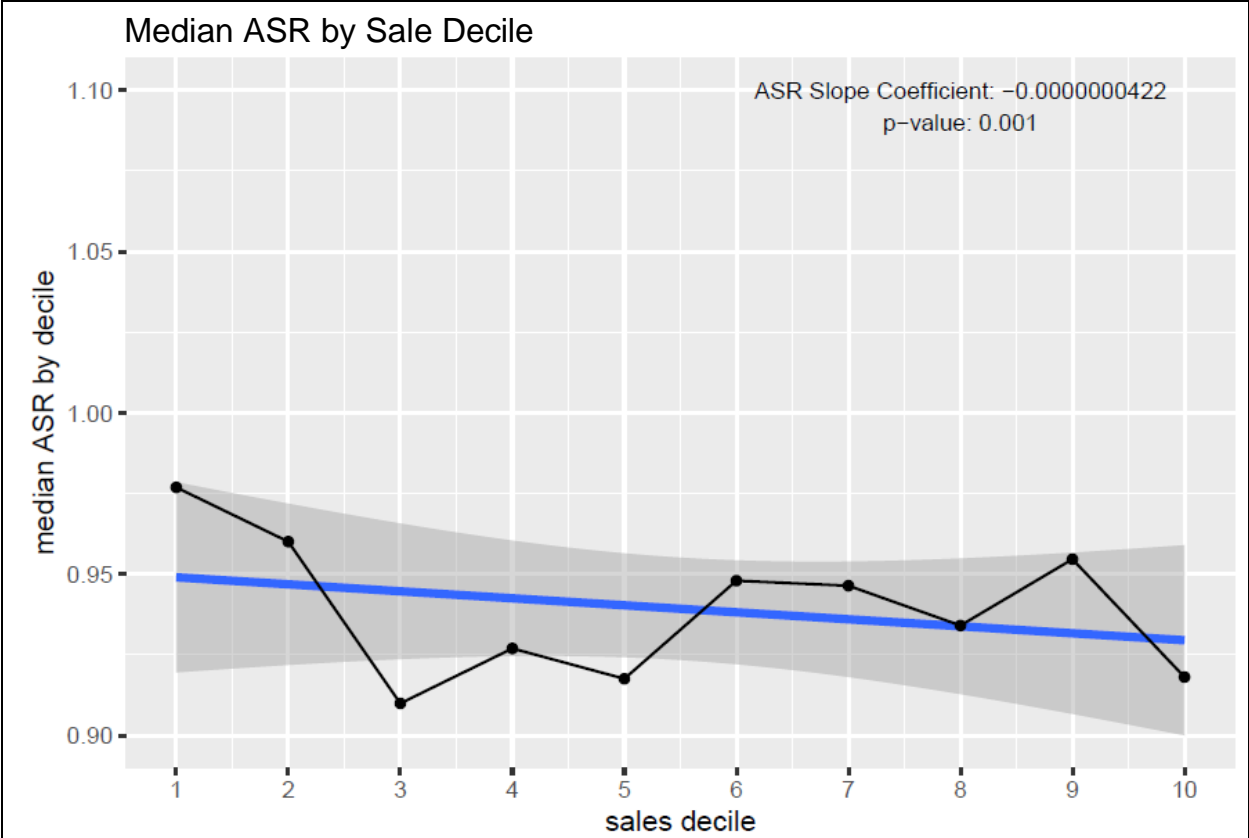
This report provides guidance on the vertical equity of the assessed values for the file that was provided. Vertical equity for assessments is defined as: “differences in the levels of assessment of properties related to the value ranges of the properties, i.e., properties of higher value have assessment levels different from properties of lower value.” Vertical equity exists when assessment levels remain consistent across the entire range of sales prices.

Detailed guidance on sales ratio studies is provided by International Association of Assessing Officers Standard for Sales Ratio Studies (Standard). Measuring the level of vertical equity of assessments is a complex task since the market value of any given property is unknown. As a result, the notion of a single or best measure of vertical equity may be illusive. Instead, the best approach may be to calculate and evaluate all the current measures in the Standard, with the addition of Ginis measures which are new to the assessment profession, to develop a more complete picture of the level of vertical equity for a given distribution. The following analysis is provided based in part on the Standard.

Assessment Ratio Analysis

The first step in this analysis is an analysis of the assessment to sale ratio which is calculated by dividing a property’s assessment by its sale price. The overall assessment level is determined by the median assessment to sale ratio (ASR). The median ratio is preferred because it is less affected by extreme values than other measures of central tendency, such as averages. **The overall median ASR is 0.939.**

The median ASR can also be used to evaluate vertical equity when calculated for each sale price decile as provided in this following chart. The chart displays how the ASR changes as the sales price increases. **The range between median ASR for the lowest priced decile (0.977) and highest (0.918) decile is 0.059.** This suggests a trend toward lower ratios as sales prices increase, a **regressive trend.**



Coefficient of Dispersion

The Coefficient of Dispersion, or COD, is not considered to be a measure of vertical equity. It does, however, provide important information on the uniformity or variability of the distribution, and therefore can be informative in this context.

The COD of the properties provided is 10.974. IAAO guidance for the COD are provided below for various residential property uses.

IAAO COD Ranges by Property Type

Type of Property - General	Type of Property - Specific	COD Range
Single-family residential (including residential condominiums)	Newer or more homogeneous areas	5.0 to 10.0
Single-family residential	Older or more heterogeneous areas	5.0 to 15.0
Other residential	Rural, seasonal, recreational, manufactured housing, 2-4 unit family housing	5.0 to 20.0
Income-producing properties	Larger areas represented by large samples	5.0 to 15.0
Income-producing properties	Smaller areas represented by smaller samples	5.0 to 20.0

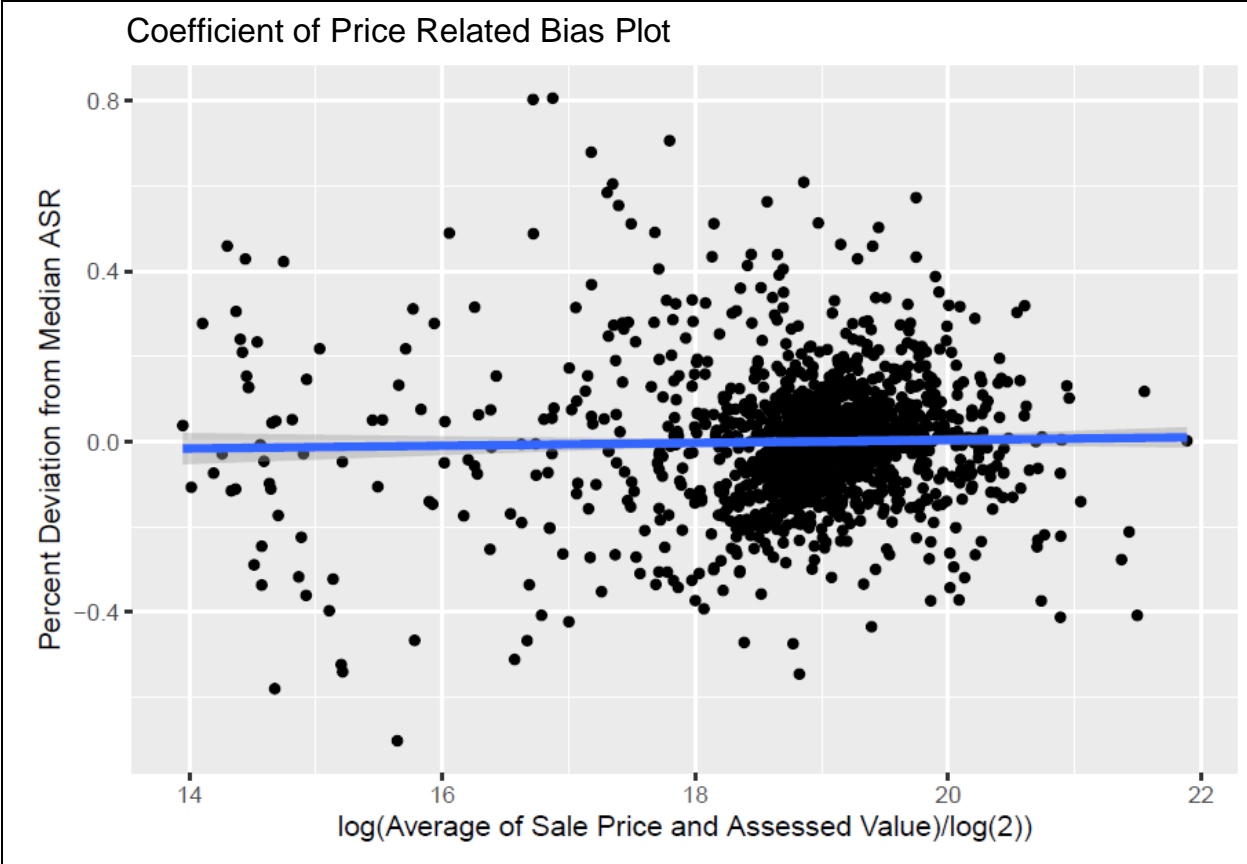
Type of Property - General	Type of Property - Specific	COD Range
Other real and personal property		Varies with local conditions

Price Related Differential

The Price Related Differential, or PRD is an index statistic used for measuring vertical equity. The PRD is calculated by dividing the mean ASR by the weighted mean ASR. If vertical equity exists, this ratio should be close to 1.00. A PRD above 1.00 suggests a regressive distribution where lower-valued properties are assessed at a higher level than higher-valued properties. A PRD below 1.00 suggests a progressive distribution, where the reverse is true. The PRD as a vertical equity measure is useful but somewhat flawed. The PRD's strengths are its simplicity and ease of calculation. A disadvantage of the PRD is that it can be distorted by a small number of very high-priced properties that can lead to an indication of regressivity when none is present. As a result, the PRD can be a useful indicator that more analysis is necessary, but by itself it is not necessarily conclusive that vertical inequity exists. **The PRD for the properties provided is 1.01, which is inside the acceptable range of .98 to 1.03 recommended by the IAAO and indicates vertical equity is present.**

Coefficient of Price Related Bias

The Coefficient of Price Related Bias, or PRB, measures vertical equity with an index calculated by regression that quantifies the relationship between ASRs and value in percentage terms. If vertical equity exists, the PRB value should be close to zero. A PRB below zero suggests a regressive distribution where lower-valued properties are assessed at a higher level than higher-valued properties. A PRB above zero suggests a progressive distribution, where the reverse is true. The PRB is a more robust measure that is not as susceptible to the influence of outliers as the PRD. **The PRB for the properties provided is 0.003, which is inside the IAAO recommended range of -0.05 – 0.05 and indicates vertical equity is present.**



Spearman’s Rank-Order Correlation

While not specifically discussed in the section of the Standard on vertical equity, the Spearman Rank test is mentioned in the Test of Hypotheses section of the Standard. Specifically, Spearman’s Rank is recommended for evaluating whether low and high prices properties are appraised at equal percentages of market value i.e., whether vertical equity exists.

The Spearman’s Rank measures the strength and direction of association between two ranked variables. Ranks are determined for each property in the analysis, one rank for sale price, the other for assessment to sale ratio (ASR). A correlation analysis of the two ranks is performed, with the resulting statistic referred to as Spearman’s rho. A Spearman’s rho of close to 0 demonstrates a low correlation between ASR and sales price and suggests vertical equity is present. A statistical test (t-test) is also performed to determine if the correlation is statistically significant. The distribution of a Spearman’s rank is also plotted to provide a visual representation of the analysis.

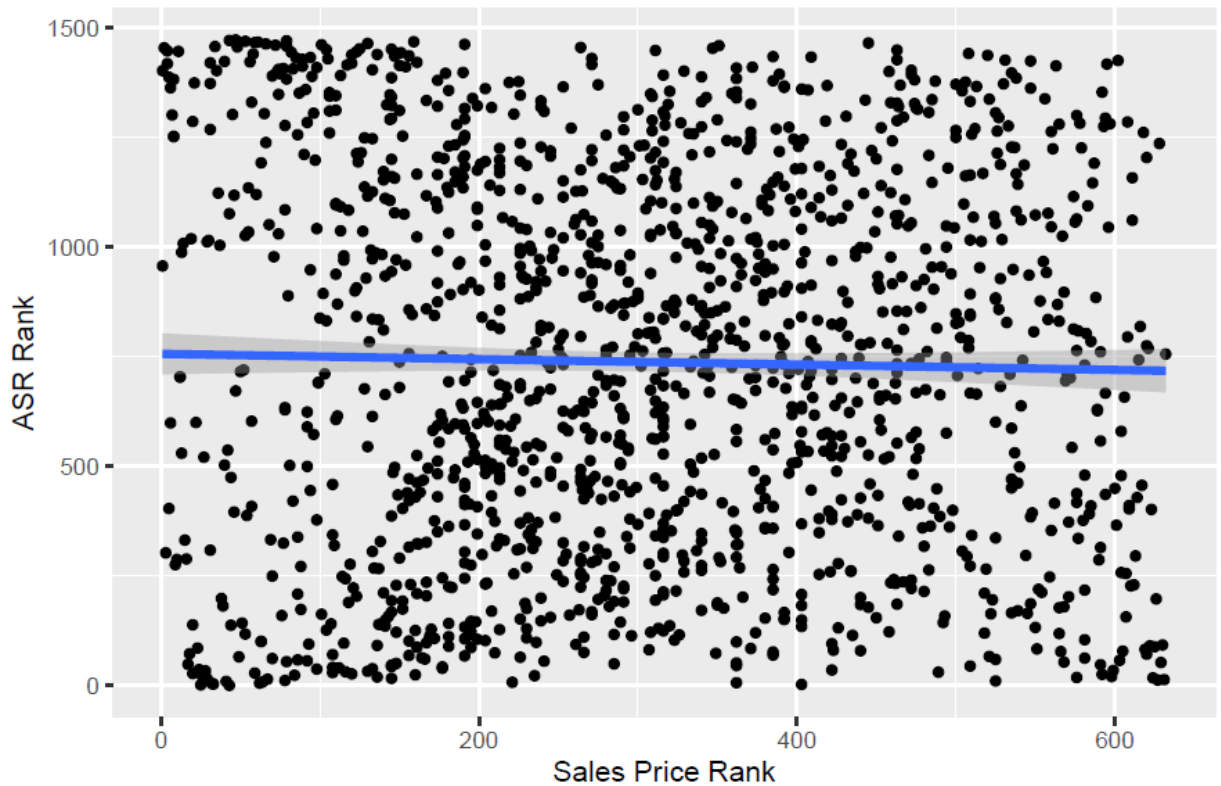
While the Spearman analysis can provide useful information on the relationship between ASR and sales, its limitation is that it is a simple correlation that lacks other important information on the distribution. **A Spearman’s Rho of -0.015 suggests a low correlation of the ASR and sales ranking, which is confirmed in the plot of these data. The asymptotic T-statistic approximation suggests this correlation is not statistically significant at a 0.553 confidence level. As such, the Spearman test indicates vertical equity.**

Spearman’s Rho Ranges

Spearman’s Rho Coefficient	Interpretation	Acceptable Range
-1.0 to -0.9	Very Strong Regressive Correlation	No
-0.89 to -0.70	Strong Regressive Correlation	No

Spearman's Rho Coefficient	Interpretation	Acceptable Range
-0.69 to -0.40	Moderate Regressive Correlation	No
-0.39 to -0.20	Weak Regressive Correlation	Yes
-0.19 to 0.01	Very Weak Regressive Correlation	Yes
0	No Correlation	Yes
0.01 to 0.19	Very Weak Progressive Correlation	Yes
0.20 to 0.39	Weak Progressive Correlation	Yes
0.40 to 0.69	Moderate Progressive Correlation	No
0.70 to 0.89	Strong Progressive Correlation	No
0.90 to 1.0	Very Strong Progressive Correlation	No

Spearman's Rank-Order Correlation Plot



Gini Measures

While the use of Gini measures as a method for measuring inequity has been common in economic analysis for decades, it has only recently been considered as a measure of vertical equity for assessments. More information on the use of Gini measures to measure vertical equity in assessments can be found in recent papers by Quintos (2020), and McMillen and Singh (2021).

Gini measures are like Spearman's rank in that they do not use sales ratios to determine equity and that both measures use a ranking of variables in their calculation. However, Ginis go beyond a simple measure of co-movement as is done in Spearman's Rank, quantifying how the distribution of assessments behave relative to the distribution of sales at given price levels. By capturing the cumulative distribution behavior of assessments and prices across ordered price levels, Gini measures provide more information on the vertical equity of the entire distribution. The Gini analysis provides two numerical measures that summarize the relationship of the sales and assessment distributions, the Kakwani index (KI) and the Modified Kakwani

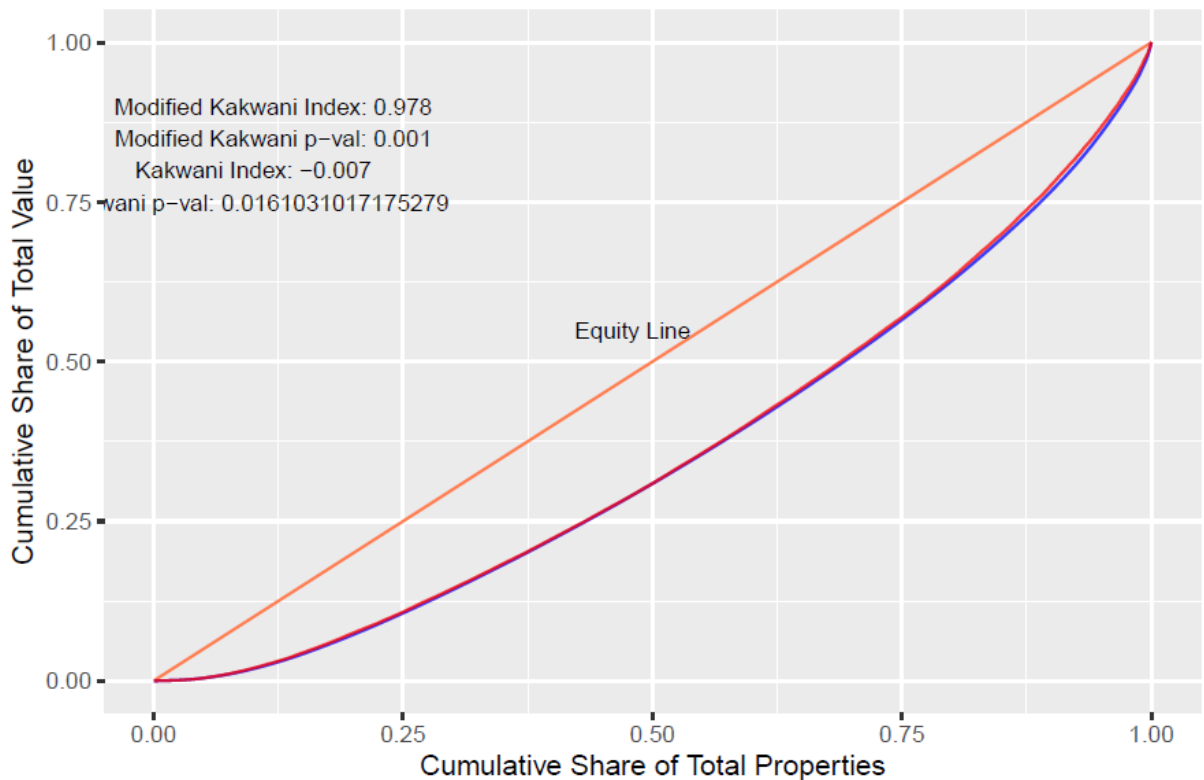
index (MKI). Assessments are considered regressive when KI is less than zero, or MKI is less than one. A progressive distribution occurs when KI is above zero, or MKI is above one.

In addition to these indexes, the level of equity in assessments can also be visualized in a Gini analysis via a plot of the relationship between of the sales and assessments. If the lines move together, vertical equity is present. Where the line representing the assessments lies above the sales line, a regressive distribution is indicated. Where the assessment line is below the sales line, a progressive distribution is implied.

Unlike the other vertical equity measures which provide a global measure of vertical equity for the entire distribution, the Gini plot provides an indicator of the measure of equity across the entire price distribution. This information can be used to better understand the performance of a valuation model across different segments of the price distribution, allowing for better diagnosis of potential vertical equity issues.

The Gini analysis for this distribution provides a **KI of -0.007** and a **MKI of 0.978** indicating **vertical equity** in the distribution. This finding is confirmed through the inspection of the lines representing assessments and sales produced by the Gini analysis.

Gini Measures Plot



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